

Exercise 5

If a ball is thrown into the air with a velocity of 40 ft/s, its height in feet t seconds later is given by $y = 40t - 16t^2$.

(a) Find the average velocity for the time period beginning when $t = 2$ and lasting

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|--------------------|-------------------|
| (i) 0.5 seconds | (ii) 0.1 seconds |
| (iii) 0.05 seconds | (iv) 0.01 seconds |

(b) Estimate the instantaneous velocity when $t = 2$.

Solution

For each value of t , calculate y and then use the slope formula to find the slope of the secant line. Note that $y = 16$ at $t = 2$.

$$t = 2 + 0.01 \quad \rightarrow \quad y \approx 15.7584 : \quad (2.01, 15.7584) \quad \Rightarrow \quad m = \frac{15.7584 - 16}{2.01 - 2} = -24.16$$

$$t = 2 + 0.05 \quad \rightarrow \quad y \approx 14.76 : \quad (2.05, 14.76) \quad \Rightarrow \quad m = \frac{14.76 - 16}{2.05 - 2} = -24.8$$

$$t = 2 + 0.1 \quad \rightarrow \quad y \approx 13.44 : \quad (2.1, 13.44) \quad \Rightarrow \quad m = \frac{13.44 - 16}{2.1 - 2} = -25.6$$

$$t = 2 + 0.5 \quad \rightarrow \quad y = 0 : \quad (2.5, 0) \quad \Rightarrow \quad m = \frac{0 - 16}{2.5 - 2} = -32$$

The slope m is the average velocity in the time interval. The instantaneous velocity at $t = 2$ is

$$\begin{aligned} \frac{dy}{dt} &= 40 - 16(2)t \\ \left. \frac{dy}{dt} \right|_{t=2} &= 40 - 16(2)(2) = -24. \end{aligned}$$